LONGITUDINAL SOLITONS IN RHIC*

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Abstract

Stable, coherent, longitudinal oscillations have been observed in the RHIC accelerator. Within the context of pertubation theory, the beam parameters and machine impedance suggest these oscillations should be Landau damped. When nonlinear effects are included, long lived, stable oscillations become possible for low intensity beams. Simulations and theory are compared with data.

1 INTRODUCTION

Solitary waves in the form of notches or hotspots have been observed in coasting beams and the theory of solitary waves in plasmas [1] and coasting beams have been discussed in [2, 3, 4, 5, 6]. As an introduction we will use a very simple model due to Sacherer[7]. Consider a coasting beam with a phase space density that is piecewise constant. Figure 1 shows a simple picture in the frame comoving with the soliton, where the phase space density is either 0, f_0, or $f_0 + f_1$; and the distribution is independent of time. We use x as the longitudinal coordinate and $p = dx/dt$. The coasting beam Hamiltonian is

$$H = p^2/2 + \ell/2 \int dp f(p, x)$$

where ℓ is negative for a focusing impedance. Since the phase space density is constant on contours of constant H one obtains algebraic equations, $H(x = 0, p = p_1) = H(x = L, p = 0)$ and $H(x = 0, p = p_0 + p_2) = H(x = L, p = p_0)$. While Sacherer resorted to numerical methods these equations are straightforward if one assumes $p_1 \ll p_0$ which results in

$$p_1 \approx \frac{-2\ell f_1}{1 + \ell f_0/p_0}.$$

If the correction term $\ell f_0/p_0$ is set to zero, the condition is identical to that for a phase space density of f_1 to self bunch. The change in the line density due to soliton is $\approx -p_1^2/\ell$. For an inductive impedance above transition $\ell > 0$ and one observes a notch, or hole in a wall current monitor (WCM) signal.

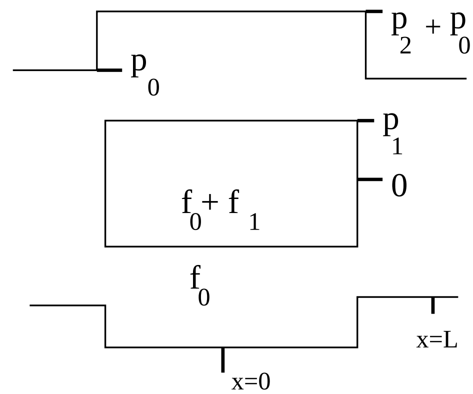

Figure 1: Simple picture of a soliton in a coasting beam. The horizontal axis is x, the longitudinal position within the bunch. The vertical axis is $p = dx/dt$.

2 DATA

Long lived coherence has been observed in the SPS[8], the Tevatron[9], and now the RHIC. Figure 2 shows a mountain range plot of the WCM for freshly injected protons with $\gamma = 25.9$. The amplitude of the coherent oscillation increased steadily, and Figure 3 shows the same bunch, still at injection energy, 17 minutes later. Figures 4 and 5 show different bunches at flattop with $\gamma = 107$. In all the cases shown, only the 28 MHz accelerating cavities were operating and the total acquisition time was 4000 turns ≈ 50 milliseconds. RHIC's transition energy is $\gamma_T = 23.8$, so all the data are above transition. All the data show a coherent oscillation which corresponds to a region of overdensity, or hot spot, in the longitudinal phase space. This behavior is commonplace in RHIC and we have never observed a stable hole. Measurements of RHIC's broad band impedance[10] give $Z/n = j(3 \pm 1)\Omega$ for the inductive wall contribution. The longitudinal space charge impedance at $\gamma = 25.9$ is $Z/n = j1.3\Omega$ and the space charge impedance becomes negligible at store. Therefore, we see hotspots with a defocusing impedance, which is just the reverse of what one expects for coasting beams.

3 THEORY

Attempts to understand the "dancing bunches" in the Tevatron[9] are based on the linearized theory of coherent instabilities[11]. The main idea is that the coherent tune shift due to the broad band impedance is larger than the synchrotron tune spread. This results in undamped coher-

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ent modes. If this was the case in RHIC, the data shown in Figure 5 would require many modes and one would expect to see all kinds of coherent oscillations for different bunch lengths and intensities. We always see one, perhaps two, hotspots. In the rest of this section we develop an alternate theory which yields stable, long lived hotspots[12].

Let ϕ denote the position of a particle in the bunch, measured in units of RF radians; $\omega_{s,0}$ denote the small amplitude angular synchrotron frequency; and use $s = \omega_{s,0}t$ as the evolution variable. Let $\rho(\phi, s)$ be the normalized line density of the particles so that $\int d\phi\rho(\phi, s) = 1$. Take a simple broad band impedance model $Z = j\omega L$. Let $V(\phi) = V_{rf}\sin\phi$ be the RF voltage and let ω_{rf} be the angular RF frequency. Note that our definitions give $V_{rf} > 0$ below transition and $V_{rf} < 0$ above transiton. Let Q denote the total charge within the bunch. Then the equation of motion for ϕ is

$$\frac{d^2\phi}{ds^2} + \sin\phi = \frac{LQ\omega_{rf}^2}{V_{rf}}\frac{\partial\rho(\phi, s)}{\partial\phi}, \tag{1}$$

To simplify notation set $\ell = -LQ\omega_{rf}^2/V_{rf}$. For a steady state, matched bunch, a positive value of ℓ defocuses the beam and leads to an incoherent synchrotron frequency that is less than the synchrotron frequency for $\ell = 0$. We have done multi-particle drift-kick simulations and have verified that equation (1) creates high density solitons for $\ell > 0$.

Equation (1) describes a Hamiltonian system, and we make a canonical transformation to the action angle variables for a simple harmonic oscillator J and Ψ. We make the ansatz that the phase space density undergoes a rigid rotation $f(J, \Psi, s) = g(J, \Psi - (1-\beta)s)$ where the coherent frequency of the soliton is $\omega_c = (1-\beta)\omega_{s,0}$. The Hamiltonian is then phase averaged over s resulting in

$$K = \beta J + \alpha(J) + V(J, \Psi), \tag{2}$$

where $\alpha(J) \approx -J^2/16$, generates detuning with synchrotron action and the coherent forces are generated by

$$V(J, \Psi) = \frac{\ell}{\pi}\int \frac{g(J_1, \Psi_1)d\Psi_1 dJ_1}{\sqrt{2J + 2J_1 - 4\sqrt{JJ_1}\cos(\Psi - \Psi_1)}}. \tag{3}$$

The Vlasov equation is

$$\frac{\partial K}{\partial J}\frac{\partial g}{\partial \Psi} - \frac{\partial K}{\partial \Psi}\frac{\partial g}{\partial J} = 0, \tag{4}$$

The simplest solutions of equation (4) are of the form $g(J, \Psi) = G(K(J, \Psi))$, without regard to separatricies. Both analytic and numerical solutions have been obtained. It is easiest to switch to Cartesian variables $A = \sqrt{2J}\sin\Psi$, $B = \sqrt{2J}\cos\Psi$. The analytic work relies on approximating the unperturbed Hamiltonian $K_0 = \beta J + \alpha(J) \approx \hat{K} - \lambda(A - A_0)^2$ where $A = A_0$, $B = 0$ is the center of the soliton; and $\lambda \approx A_0^2/16$. Consider a phase space density of the form

$$g(A, B) = \frac{3}{2\pi ab}\sqrt{1 - (A - A_0)^2/a^2 - B^2/b^2} \tag{5}$$

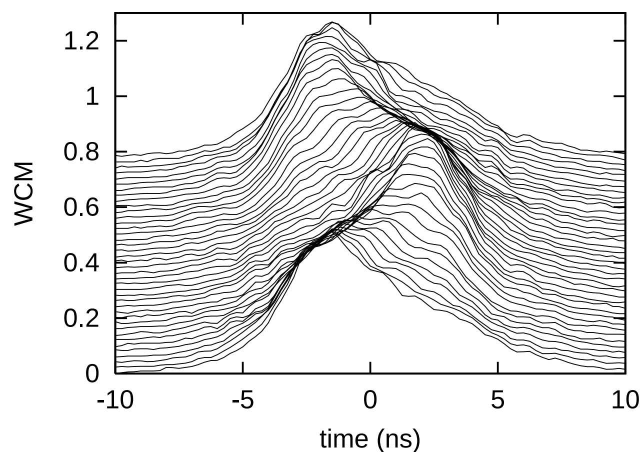

Figure 2: WCM data for a freshly injected bunch

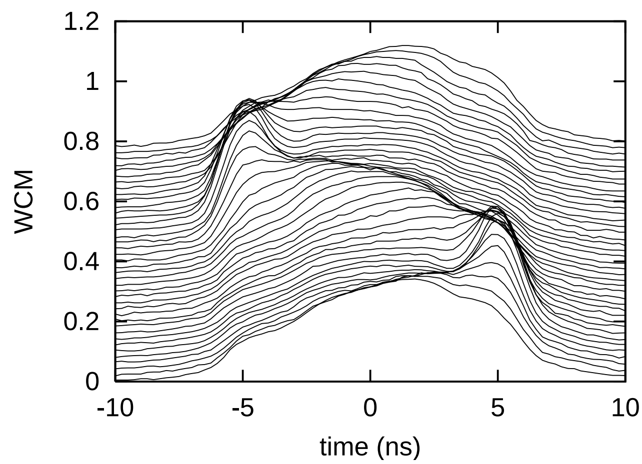

Figure 3: WCM data at injection for the same bunch as in Figure 2, but 17 minutes later.

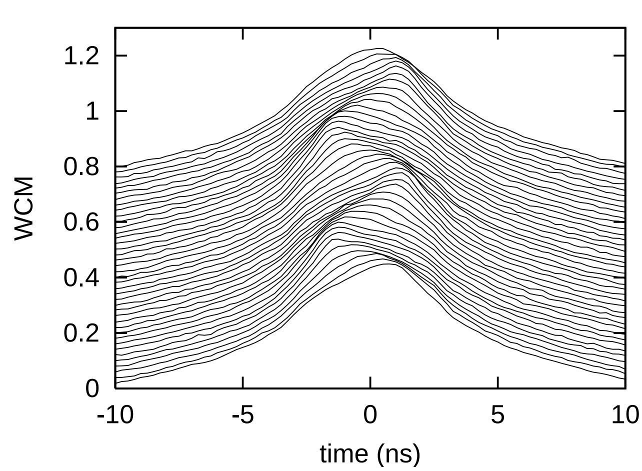

Figure 4: WCM data for a bunch at the beginning of flattop

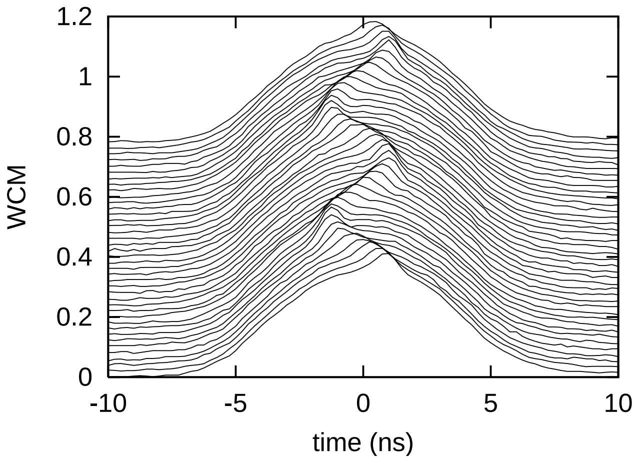

Figure 5: WCM data for a different bunch at flattop.

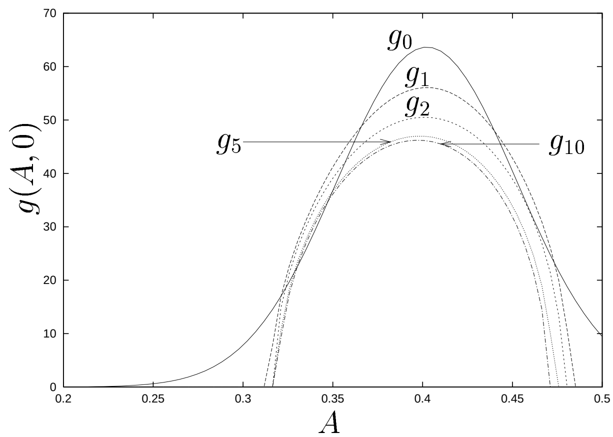

Figure 6: Values of the distribution taken through the line containing the coordinate origin and the peak of the soliton, during the interative solution.

where a and b are the half widths of an ellipse and the solution is zero outside this ellipse. With this density $\partial V/\partial A = C_A(A - A_0)$ and $\partial V/\partial B = C_B B$, for points inside the ellipse. When combined with the previous approximation, the Vlasov equation is solved if

$$a^2(C_A - 2\lambda) = b^2 C_B. \tag{6}$$

Letting $r = a/b$ one finds that $0 \le r < 1$ and

$$\frac{a^3\lambda}{\ell} = \sum_{n=0}^{\infty}\left[\frac{(2n-1)!!}{2^n n!}\right]^2 (1-r^2)^n\left[\frac{3r}{4} - \frac{3nr^3}{1-r^2}\right], \tag{7}$$

$$\equiv R(r) \approx -0.464 r\ln r - 0.285 r(1-r) \tag{8}$$

where $(-1)!! \equiv 1$, $0 \le R \le 0.10$, and the fractional error of the approximate expression is $\le 5\%$. Positive values of ℓ are needed for self bunching. Also, since $a < b$, the peak of the line density is largest when the soliton is farthest from the bunch center, as in Figure3.

We have also done iterative solutions. We search for solutions of the form $g(A, B) = G(K(A, B))$. Start by choosing a value of β and take an initial distribution $g_0(A, B)$. Iterate using $g_{n+1}(A, B) = G(K_0(A, B) + V_n(A, B))$, where $V_n(A, B)$ is calculated using equation (3) with g_n. Figures 6 and 7 show solutions for $G(K) = C_0\theta(K - K_0)\sqrt{K - K_0}$ which is the same as was used for the analytic solution.

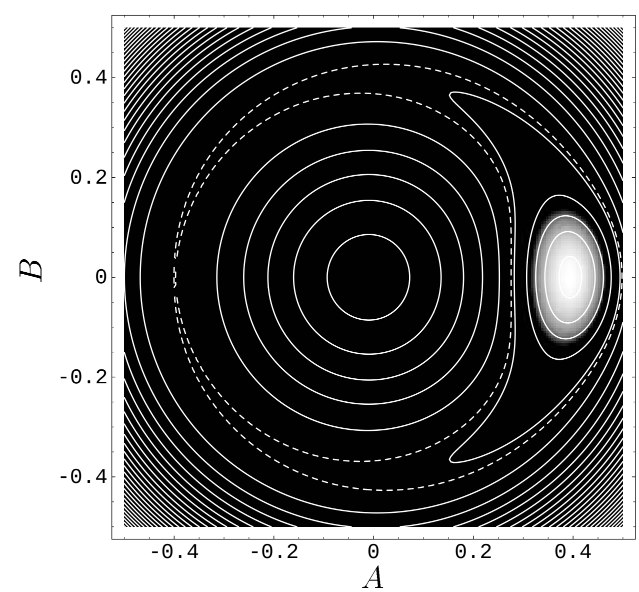

Figure 7: Final solution after 10 iterations.

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